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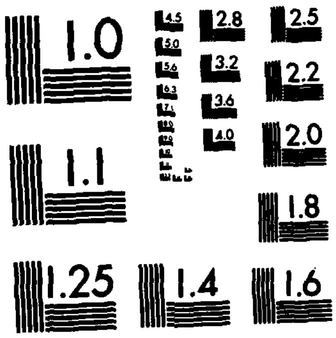

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MELBOURNE, VICTORIA

AERODYNAMICS NOTE 406

DIGITAL FILTERING OF HELICOPTER FLIGHT DATA

by

N. E. GILBERT and J. A. FLEMING

Approved for Public Release

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SUMMARY

Data obtained during flight trials on a Sea King Mk. 50 helicopter contain significant noise, especially in measurements of components of linear acceleration and angular velocity. Spectral analysis of representative flight data show significant unwanted signal components to be mostly at the rotor frequency and higher harmonics, especially the fifth, which is the blade passing frequency. For channels requiring filtering, two suitable digital Butterworth lowpass filters are designed and their effect demonstrated. Direct representation of the transfer function as a high-order filter is used, in preference to representation as a number of second-order and first-order sub-filters. By using double precision to overcome the usual problems of numerical accuracy with the direct form, the advantages of simplicity and economy of programming effort are realized. For a general-order filter, the mathematical derivation of the filter coefficients in direct form is given.

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16. Abstract

Data obtained during flight trials on a Sea King Mk. 50 helicopter contain significant noise, especially in measurements of components of linear acceleration and angular velocity. Spectral analysis of representative flight data show significant unwanted signal components to be mostly at the rotor frequency and higher harmonics, especially the fifth, which is the blade passing frequency. For channels requiring filtering, two suitable digital Butterworth lowpass filters are designed and their effect demonstrated. Direct representation of the transfer function as a high-order filter is used, in preference to representation as a number of second-order and first-order sub-filters. By using double precision to overcome the usual problems of numerical accuracy with the direct form, the advantages of simplicity and economy of programming effort are realized. For a general-order filter, the mathematical derivation of the filter coefficients in direct form is given.

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NOMENCLATURE

A	Attenuation (dB)
A_1, A_2	Value of A at f_1, f_2
$A_p(r)$	$(-1)^p C_p^r$
$B_q(r)$	C_q^{N-r}
C_p^N	Binomial coefficient, $= N!/(N-p)!p!$
$D_p(r)$	$\sum_{q=0}^{p} A_q(r) B_{p-q}(r)$
$E_0, E_1, \ldots, E_N$	Numerator coefficients of digital filter transfer function
$F_1, F_2, \ldots, F_N$	Denominator coefficients of digital filter transfer function
$H(s)$ or $H(j\Omega)$	Transfer function of analogue filter
$H(z)$ or $H(e^{j\omega T})$	Transfer function of digital filter, $= Y(z)/X(z)$
N	Order of filter
T	Sampling period
$X(z)$, $Y(z)$	z-transform of x_N, y_N
a_p	$\Omega_c^{-p} \prod_{m=1}^{p} \{\cos[(m-1)\pi/2N]/\sin(m\pi/2N)\}$ for $p = 1, 2, \ldots, N$; $(a_0 = 1)$
a_1, a_2	$\sum_{p=0}^{N} E_p \cos(p\omega T)$, $\sum_{p=0}^{N} E_p \sin(p\omega T)$
b_1, b_2	$\sum_{p=0}^{N} F_p \cos(p\omega T)$, $\sum_{p=0}^{N} F_p \sin(p\omega T)$
f	Digital frequency (Hz)
f_c	Digital cut-off frequency (Hz)
f_1, f_2	Specified values of f in passband, stopband
j	$(-1)^{1/2}$
k_0, k_1	Constants used to test for a drop-out
m, p, q, r	Summation integers
s	Laplace transform variable
s_{2p+1}	Poles of $H(s)$ in left-half s plane for $p = 0, 1, \ldots, N-1$
$x_0, x_1, \ldots, x_N$	Unfiltered sequence of data
$y_0, y_1., \ldots, y_N$	Filtered sequence of data
z	z-transform variable
Ω	Analogue frequency (rad/s)
Ω_c	Analogue cut-off frequency (rad/s)

$\tau(e^{j\omega T})$	Phase, or time, delay of filter
$\phi(e^{j\omega T})$	Phase of filter
ω	Digital frequency (rad/s), $=2\pi f$
ω_c	Digital cut-off frequency (rad/s), $=2\pi f_c$
ω_1, ω_2	Value of ω at f_1, f_2

1. INTRODUCTION

In 1979 the Aeronautical Research Laboratories (ARL) conducted flight trials on a Royal Australian Navy Sea King Mk. 50 helicopter. The aim of the trials was to record data of good quality which could be used to validate a mathematical model developed by ARL [1,2]. The data acquisition system installed at the time of the trials allowed 32 channels of data to be recorded on magnetic tape in serial digital form, at sampling frequencies of 60, 30, or 15 Hz [3]. Despite the inclusion of an analogue Butterworth lowpass filter on most channels of recorded data, it was found that there was significant noise remaining in a number of these channels, especially those measuring components of linear acceleration and angular velocity. It was decided therefore to design and implement digital Butterworth lowpass filters that could be used, optionally, to remove noise from any channel. This type of filter was selected because of its good (i.e. close to ideal) amplitude response, coupled with a reasonably linear phase response, at lower frequencies in the passband [4].

The digital transfer function is usually represented either as a series (i.e. cascade) or parallel, or combination of both, of second-order and first-order sub-filters (Ref. 5, p. 44; Ref. 6, p. 26; Ref. 7, p. 43). Direct representation as a high-order filter is generally avoided for reasons of numerical accuracy. However, the direct form is much simpler to program and double precision may be used to retain sufficient accuracy. With recent trends in computing towards efficiency and simplicity in programming effort at the possible expense of CPU time, it was decided to use the direct form. Because no reference could be found in which the filter coefficients are derived for a general-order filter in the direct form, the lengthy algebraic process is included in Section 2 in a description of the basic theory used to design the digital filter.

In Section 3, flight data recorded for a particular manoeuvre, together with power spectra, are presented for most channels that were properly functioning at the time of recording. In obtaining power spectra, the low frequency signal components are first removed to accentuate unwanted frequency components at and above the one-per-rev rotor frequency of 3·5 Hz. In Section 4, the filtering procedure adopted for each channel is given. In cases where the control input and responses to it have a substantial energy content at relatively high frequencies, the analogue filtering may cause some distortion of the wanted signal. Hence, care should be taken to avoid further distortion when using a digital filter. For this reason and because of the absence of significant noise, control inputs (i.e. pilot stick and pedal displacements) and displacement quantities directly linked (i.e. mechanically or electrically rather than aerodynamically) to these inputs are not filtered. For all channels requiring filtering, two standard fifth-order filter specifications are given. One has a high attenuation of 50 dB at the rotor frequency of 3·5 Hz, giving a low cut-off frequency of 1·12 Hz, and removes noise at and above the rotor frequency. It is suitable for variables where the energy is confined to relatively low frequencies, and its effectiveness is demonstrated on three different types of signal. The other filter, which has a higher cut-off frequency of 4 Hz, is used alternatively where the high frequency content is significant. Generally, of the kinematic quantities measured, the only one where this is likely to be the case is vertical acceleration when there is a violent manoeuvre. Hence, for such a manoeuvre, the effect of both filters on the vertical acceleration signal is shown.

2. DIGITAL FILTER DESIGN

2.1 General Realization of a Recursive Filter

For a recursive realization of a digital filter transfer function, the functional relationship between the input sequence $x_0, x_1, \ldots, x_N$ and the resulting output sequence $y_0, y_1, \ldots, y_N$ is given by the difference equation (Ref. 6, p. 22; Ref. 7, p. 40; Ref. 8, p. 197)

$$\sum_{p=0}^{N} E_p x_{N-p} = \sum_{p=0}^{N} F_p y_{N-p} \qquad (1)$$

where $E_0, E_1, \ldots, E_N$ and $F_0, F_1, \ldots, F_N$ are the filter coefficients. Rewriting Equation (1) to solve for y_N explicitly,

$$y_N = \frac{1}{F_0}\left[\sum_{p=0}^{N} E_p x_{N-p} - \sum_{p=1}^{N} F_p y_{N-p}\right]. \tag{2}$$

The digital filter is then realized in terms of the constant filter coefficients, which must be determined.

In digital filter theory, it is convenient to transform the above time-domain equation to the z-domain using the z-transform, in which each uniform time delay in both input and output sequences is represented by a multiplication by z^{-1} (Ref. 8, p. 45). Hence

$$Y(z) = \frac{1}{F_0}\left[\sum_{p=0}^{N} E_p z^{-p} X(z) - \sum_{p=1}^{N} F_p z^{-p} Y(z)\right] \tag{3}$$

where $X(z)$ and $Y(z)$ are the z-transforms of x_N and y_N respectively. From Equation (3), the transfer function from which the digital filter may be realized is given by

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{p=0}^{N} E_p z^{-p}}{\sum_{p=0}^{N} F_p z^{-p}} \tag{4}$$

2.2 Butterworth Digital Filter Coefficients

The squared magnitude response for an analogue Butterworth lowpass filter is of the form (Ref. 8, p. 211)

$$|H(j\Omega)|^2 = \frac{1}{1 + (j\Omega/j\Omega_c)^{2N}} \tag{5}$$

where Ω is analogue frequency in rad/s, Ω_c is the *cut-off frequency* (i.e. amplitude response is 3 dB down at $\Omega = \Omega_c$), and N is the order of the filter. Section 2.3 shows how both Ω_c and N, which are assumed known in this section, may be determined from a design specification.

On substituting $s = j\Omega$ in Equation (5),

$$H(s)H(-s) = \frac{1}{1 + (s/j\Omega_c)^{2N}} \tag{6}$$

The poles of Equation (6) are then at equally spaced points on a circle of radius Ω_c in the s-plane. By associating the poles in the left-half s-plane with $H(s)$,

$$H(s) = \frac{\Omega_c^N}{\prod_{p=0}^{N-1}(s - s_{2p+1})} \tag{7}$$

where (Ref. 9, p. 494; Ref. 10, p. 183)

$$s_{2p+1} = \Omega_c e^{j(2p+1+N)\pi/2N} \qquad p = 0, 1, \ldots, N-1$$

Alternatively, Equation (7) may be written as (Ref. 9, p. 494)

$$H(s) = \frac{1}{\sum_{p=0}^{N} a_p s^p} \tag{8}$$

where

$$a_p = \Omega_c^{-p} \prod_{m=1}^{p}\left\{\frac{\cos\,[(m-1)\pi/2N]}{\sin\,(m\pi/2N)}\right\} \qquad p = 1, 2, \ldots, N$$

$$a_0 = 1$$

To form the digital transfer function $H(z)$, the bilinear transformation is applied in which (Ref. 8, p. 207)

$$s = \frac{2(1 - z^{-1})}{T(1 + z^{-1})} \tag{9}$$

is substituted in Equation (8) to give

$$H(z) = \frac{1}{\sum_{p=0}^{N} a_p \left[\frac{2(1 - z^{-1})}{T(1 + z^{-1})} \right]^p} \tag{10}$$

where T is the sampling period. Expansion of Equation (10) in the direct form of Equation (4) is generally avoided because the effect of round-off errors in the filter coefficients becomes increasingly significant for higher-order filters (Ref. 7, p. 41). Therefore, the usual method for expanding Equation (10) is to write the expression in either series (cascade) or parallel form in terms of second-order and first-order sections (Ref. 5, p. 44; Ref. 6, p. 26; Ref. 7, p. 43). However, the direct form of Equation (4) is much simpler to program and the problem of coefficient sensitivity can be overcome by using double precision arithmetic in the computer program where appropriate. For overall efficiency, the direct form was chosen, and because no reference could readily be found in which Equation (10) is expanded in direct form for a general value of N, the lengthy algebraic process is now given.

Rewriting Equation (10),

$$H(z) = \frac{(1 + z^{-1})^N}{\sum_{p=0}^{N} \{a_p(z/T)^p(1 - z^{-1})^p(1 + z^{-1})^{N-p}\}} \tag{11}$$

Using the Binomial theorem to expand powers of $(1 + z^{-1})$ and $(1 - z^{-1})$,

$$H(z) = \frac{\sum_{p=0}^{N} z^{-p} C_p^N}{\sum_{r=0}^{N} \{a_r(2/T)^r \sum_{p=0}^{r} [(-1)^p z^{-p} C_p^r] \sum_{q=0}^{N-r} [z^{-q} C_q^{N-r}]\}} \tag{12}$$

where

$$C_p^N = \frac{N!}{(N-p)!\,p!} \qquad 0 \leqslant p \leqslant N$$
$$= 0 \qquad \text{otherwise}$$

It should be noted that the summation variables in the denominator of Equation (12) are rearranged at this stage in order to arrive at the final form of $H(z)$ with the desired summation variable p.

Defining

$$A_p(r) = (-1)^p C_p^r$$
$$B_q(r) = C_q^{N-r}$$

then in the denominator of Equation (12),

$$\sum_{p=0}^{r} [(-1)^p z^{-p} C_p^r] \sum_{q=0}^{N-r} [z^{-q} C_q^{N-r}] = \sum_{p=0}^{r} [z^{-p} A_p(r)] \sum_{q=0}^{N-r} [z^{-q} B_q(r)]$$
$$= [z^{-0}A_0(r) + z^{-1}A_1(r) + \ldots + z^{-r}A_r(r)][z^{-0}B_0(r) + z^{-1}B_1(r) + \ldots + z^{-(N-r)}B_{N-r}(r)]$$
$$= z^{-0}[A_0(r)B_0(r)] + z^{-1}[A_0(r)B_1(r) + A_1(r)B_0(r)] + \ldots + z^{-N}[A_0(r)B_N(r) + A_1(r)B_{N-1}(r) + \ldots + A_N(r)B_0(r)]$$
$$= \sum_{p=0}^{N} z^{-p} D_p(r)$$

where

$$D_p(r) = \sum_{q=0}^{p} A_q(r) B_{p-q}(r)$$

In the above expansion, terms are included for convenience that may be zero, e.g. $B_N(r) = 0$ if $r > 0$.

The denominator of Equation (12) is then given by

$$\sum_{r=0}^{N} \{a_r(2/T)^r \sum_{p=0}^{N} z^{-p} D_p(r)\} = a_0(2/T)^0[z^{-0}D_0(0) + z^{-1}D_1(0) + \ldots + z^{-N}D_N(0)]$$
$$+ a_1(2/T)^1[z^{-0}D_0(1) + z^{-1}D_1(1) + \ldots + z^{-N}D_N(1)]$$
$$+ \ldots + a_N(2/T)^N[z^{-0}D_0(N) + z^{-1}D_1(N) + \ldots + z^{-N}D_N(N)]$$

$$= \sum_{p=0}^{N} \{z^{-p} \sum_{r=0}^{N} a_r (2/T)^r D_p(r)\}$$

Hence, Equation (12) may be written as

$$H(z) = \frac{\sum_{p=0}^{N} z^{-p} C_p^N}{\sum_{p=0}^{N} \left\{ z^{-p} \sum_{r=0}^{N} a_r (2/T)^r D_p(r) \right\}} \tag{13}$$

which is the direct form of Equation (4) with the filter coefficients given by

$$E_p = C_p^N$$

$$F_p = \sum_{r=0}^{N} a_r (2/T)^r D_p(r)$$

Fortran subroutines for obtaining these coefficients are listed in Appendix A. Only the denominator coefficients need be calculated using double precision, since the numerator coefficients are integer.

2.3 Design Specification

For a digital Butterworth lowpass filter, defined in the form of Equation (13), values of the parameters Ω_c and N that enable the desired specifications to be met need to be determined. Frequently, these design specifications are given in terms of a maximum attenuation A_1 (dB) at a frequency f_1 (Hz) in the passband and a minimum attenuation A_2 (dB) at a frequency f_2 (Hz) in the stopband (Ref. 8, p. 214). To correct for distortion inherent in using the bilinear transformation, the specified digital frequencies f_1 and f_2 in Hz, after first converting to radian frequencies ω_1 and ω_2, must be prewarped to the corresponding analogue frequencies Ω_1 and Ω_2 in rad/s. For general values f, ω, and Ω, this is achieved using the relations (Ref. 8, p. 208)

$$\omega = 2\pi f \tag{14}$$

$$\Omega = \frac{2}{T} \tan\left(\frac{\omega T}{2}\right) \tag{15}$$

From the definition of attenuation A in dB,

$$A = -20 \log_{10} |H(j\Omega)| \tag{16}$$

Substituting for $H(j\Omega)$ in Equation (16) from Equation (5),

$$\Omega_c = \Omega(10^{A/10} - 1)^{-1/2N} \tag{17}$$

On using Equation (17) at the two specified frequency values, Ω_c may be eliminated from the resulting two simultaneous equations to give

$$N = \frac{\log_{10} [(10^{A_1/10} - 1)/(10^{A_2/10} - 1)]}{2 \log_{10} (\Omega_1/\Omega_2)} \tag{18}$$

The value of N given by Equation (18) is then rounded up to the nearest integer and Ω_c is obtained from either simultaneous equation using this value of N.

Unless the value of N obtained from Equation (18) is already an integer, only one of the design points may be matched precisely. The particular simultaneous equation used depends on which of the design specifications one wishes to match. It has been found useful to be able to choose either of these points as a means of identifying the filter with its main purpose. For example, one may wish to identify the passband frequency as the cut-off frequency (i.e. $f_1 = f_c$ at $A_1 = 3$ dB), or the stopband frequency as the rotor frequency (e.g. $A_2 = 50$ dB at $f_2 = 3{\cdot}5$ Hz). Therefore, the above procedure is used only to obtain an initial estimate for N. The filter is then specified in terms of a selected value of N and an attenuation at a specified frequency anywhere in the frequency domain. Equation (17) then yields Ω_c directly.

2.4 Phase Delay

Ideally, it would be desirable to completely eliminate the non-linear phase response of the Butterworth filter from the filtered data. One way of achieving this is to process the data in both a forward and backward direction using the same digital filter (Ref. 11, p. 194), which results in exact cancelling of the two phase shifts. When using this technique, allowance should be made for the squaring of the transfer function. Preliminary investigations of data to be filtered showed that this added complication was not warranted, since adequate correction could be made by assuming linear phase shift and hence constant phase, or time, delay. Given the transfer function $H(z)$, an expression giving this correction is now derived.

In the analogue filter, frequency is measured along the imaginary axis in the complex s-plane, whereas, in the digital filter, frequency is measured along the circumference of the unit circle in the z-plane (Ref. 5, p. 43). Hence, on substituing $z = e^{j\omega T}$ in Equation (4),

$$H(e^{j\omega T}) = \frac{\sum_{p=0}^{N} E_p e^{-pj\omega T}}{\sum_{p=0}^{N} F_p e^{-pj\omega T}} = \frac{a_1 - ja_2}{b_1 - jb_2} \tag{19}$$

where

$$a_1 = \sum_{p=0}^{N} E_p \cos(p\omega T) \qquad a_2 = \sum_{p=0}^{N} E_p \sin(p\omega T)$$

$$b_1 = \sum_{p=0}^{N} F_p \cos(p\omega T) \qquad b_2 = \sum_{p=0}^{N} F_p \sin(p\omega T)$$

Both the amplitude and phase response are readily obtained from Equation (19), with the phase response $\phi(e^{j\omega T})$ given by

$$\phi(e^{j\omega T}) = \arctan\left(\frac{a_1 b_2 - a_2 b_1}{a_1 b_1 + a_2 b_2}\right) \tag{20}$$

The phase delay $\tau(e^{j\omega T})$ is then, in units of seconds (Ref. 12, p. 66)

$$\tau(e^{j\omega T}) = -\phi(e^{j\omega T})/\omega \tag{21}$$

For the digital Butterworth lowpass filter, the phase response is reasonably linear at lower frequencies in the passband. Hence, the phase delay, which is assumed constant, is evaluated at $\omega = \omega_c/10$.

3. SPECTRAL ANALYSIS OF FLIGHT DATA

The data acquisition system (DAS) used for the flight trials [3] allowed, without sub-multiplexing, 20 channels of data, with 12-bit word size, to be recorded at a sampling frequency of 60 Hz. However, by sub-multiplexing each of three channels to record two separate quantities at 30 Hz, and each of another three to record four separate quantities at 15 Hz, a total of 32 channels was actually recorded. Signals are recorded from the channels in a regular sampling order at equi-spaced time increments throughout each complete sampling time interval of 1/60th second. However, it is assumed that all signals recorded within this sampling interval are at the clock time recorded at the beginning of the interval. Hence, a maximum relative time delay error of close to 1/60th second is tolerated. For the sub-multiplexed channels, the particular channel being recorded at each complete sampling interval is determined by the two least significant bits of the clock time word [3]. Values for channels not recorded at a specific time, because of sub-multiplexing, are obtained by linear interpolation, which allows a common sampling frequency of 60 Hz to be assumed when filtering the data.

Having converted the data to a format suitable for input to a DEC PDP-10 computer, the first stage in the data refinement process is to correct for drop-outs, generally caused by a misplaced significant bit in a 12-bit binary word. For an unfiltered sequence of signals x_o, x_1, x_2, which includes interpolated values for sub-multiplexed channels, a drop-out is considered to have occurred when

$$|x_2 - x_1| > k_o + k_1|x_1 - x_o|$$

where k_o and k_1 are constants determined appropriately by inspection of the flight data. For a

12-bit word, giving an integer range of 0 to 4095, values found appropriate were $k_o = 100$ and $k_1 = 5$. When a drop-out occurs, the current value is reset to its previous values, i.e. $x_2 = x_1$.

The next stage is to correct for phase shift introduced by the use of analogue Butterworth lowpass filters in the DAS. Sixth-order filters were used, with cut-off frequencies of 3, 6, and 12 Hz, generally corresponding to the sampling frequencies of 15, 30, and 60 Hz. A description of the quantities measured, together with their sampling frequency and analogue filter cut-off frequency, are given in Table 1. For the purpose of obtaining data suitable for validation of the

TABLE 1

Quantities Measured by Data Acquisition System

Channel number	Quantity measured	Sampling frequency (Hz)	Analogue filter cut-off frequency (Hz)
1	Cyclic stick position—pitch	60	12
2	Cyclic stick position—roll	60	12
3	Collective stick position	60	12
4	Angle of attack	60	12
5	Fore–aft push–pull rod position	60	3†
6	Lateral push–pull rod position	60	12
7	Collective push–pull rod position	60	12
8	Pitch rate	60	12
9	Sideslip angle	60	12
10	Roll attitude	60	12
11	Roll rate	60	12
12	Longitudinal acceleration	60	12
13	Lateral acceleration	60	12
14	Vertical acceleration	60	12
15	Pitch attitude	30	6
16	Yaw pedal position	30	6
17	Yaw rate	30	6
18*	Yaw attitude	30	—
19	Lateral cable angle	30	6
20	Longitudinal cable angle	30	6
21	Longitudinal velocity	15	6
22	Lateral velocity	15	6
23	Dynamic pressure	15	12
24	Radio altitude (raw)	15	3
25	Radio altitude (smooth)	15	3
26	Absolute pressure	15	3
27	Yaw push–pull rod position	15	12†
28	Ambient temperature	15	—
29	Torque—Engine 1	15	—
30	Rotor r.p.m.	15	—
31	Towed probe dynamic pressure	15	3
32	Towed probe differential pressure	15	3

* Switch selectable alternative measurement of "Torque—Engine 2".
† Filters inadvertently interchanged.
Note: Acceleration measurements include gravitational effects.

Sea King mathematical model [1,2], accurate correction of phase shift is considered necessary only at lower frequencies in the passband of each filter. In this region, the filters used have a reasonably linear phase response (Ref. 12, p. 112) so that a constant time delay may be assumed. Consistent with inaccuracies already accepted for relative time delays, these corrections are expressed in terms of complete sampling time intervals. For the filters with cut-off frequencies 3, 6, and 12 Hz, the corresponding time delays are 12, 6, and 3 sampling intervals.

Flight data obtained for a particular manoeuvre (ARL Sea King Flight Record No. 19100) are presented in Figures 1 to 3, in which the data is corrected for analogue filter phase delay and converted to suitable engineering units using appropriate calibration factors and offsets. The channels are grouped into three types, which include:

(1) control inputs and directly linked displacements;

(2) angular displacements and velocities; and

(3) linear velocities and accelerations, and miscellaneous quantities.

Because of a temporary fault in the recording system, sideslip angle was not measured for this particular flight, but in other flights, the characteristics of the signal were found to be similar to those of the angle of attack signal. The incremental plotter, used to present the data in Figures 1 to 3, plots in increments of 0·01 inch in both X and Y directions. Hence to avoid distortion of the signal along the time scale caused by rounding equi-spaced intervals of 1/60th second to the nearest plot increment, the scale is chosen such that each sampling interval is an integer multiple of the plot increment (in this case, the integer multiple is one, i.e. 100 samples per inch). Though the duration of the recorded flight data was 18·667 s, only the first 8·333 s (i.e. 500 sampling intervals) are shown.

In order to determine the relative energy levels in the signals for frequencies up to the Nyquist frequency, which is half the sampling frequency, spectral analysis is used [13]. In using the Fast Fourier Transform (FFT) method to obtain the Fourier coefficients [14], the number of sample values is required, by convention, to be a power of two. The quantity of data available for the flight (1121 samples), allows 2^{10} ($=1024$) samples to be analyzed when the sampling frequency is 60 Hz. Since, for sub-multiplexed channels, interpolated values are not true sample measurements, they should not be used. Hence, the corresponding number of samples at 30 and 15 Hz are 2^9 ($=512$) and 2^8 ($=256$). The signal components at low frequencies up to about 1 Hz are first removed by subtracting, from the unfiltered data, values obtained using the low cut-off frequency filter eventually adopted (see Section 4). This accentuates unwanted frequency components at and above the rotor frequency of 3·5 Hz. The noise power spectrum is then obtained by first smoothing with a Hanning window and then smoothing further with weights 0·16, 0·68, and 0·16 as defined by Blackman (Ref. 15, p. 136).

For the control inputs and directly linked displacements in Figure 4, there is an absence of significant noise. In Figures 5 and 6, significant noise is indicated for

(*a*) yaw attitude at 3·5 Hz;

(*b*) roll rate and all three accelerations* at 3·5 Hz and higher harmonics (including fifth, which is blade passing frequency of 17·5 Hz);

(*c*) pitch rate at 17·5 Hz;

(*d*) angle of attack, dynamic pressure, and torque at unspecified frequencies (i.e. random noise); and

(*e*) rotor r.p.m. at 3·5 Hz and higher frequencies.

Pitch attitude, roll attitude, yaw rate, longitudinal velocity, and lateral velocity do not have significant noise.

---

* Though the energy level for vertical acceleration is very high at 17·5 Hz, there is still significant noise at 3·5 Hz.

Repetition of the spectral analysis procedure on other flight manoeuvres produced very similar results. Hence it was considered appropriate to draw general conclusions on filtering of each channel from the trial analysed here.

4. FILTERING OF FLIGHT DATA

In adopting a filtering procedure for each channel, one must first decide on the relative merits of filtering the channel. In cases where the high frequency energy content is significant, filtering is likely to distort the signal to some degree. This is because the filtering does not distinguish between wanted and unwanted signal components.

Control inputs and directly linked displacements frequently contain substantial high frequency energy, but the noise component is generally insignificant. Hence, these channels are not filtered. Except in special cases, such as violent manoeuvres, the high frequency energy content of other channels primarily arises from noise. Since most noise is at and above the rotor frequency of 3·5 Hz, it was decided therefore to filter all these channels, including those with little noise, using a fifth-order filter with an attenuation of 50 dB at 3·5 Hz, giving a low cut-off frequency of 1·12 Hz. The effect of the filter on three types of signal is shown in Figure 7, where, for clarity, the filtered signals are offset from the unfiltered ones. It can be seen that the filter copes well with the first two types shown, where noise is clearly identifiable at specific frequencies, as well as the third type, which has random noise.

Examples of violent manoeuvres include engine cuts and sudden large movements in the collective stick, which generally result in sudden large changes in vertical acceleration, altitude, torque, and rotor r.p.m. For such cases, when filtering is necessary, a fifth-order filter is specified with a higher cut-off frequency of 4 Hz. The effect of both filters on the vertical acceleration signal is shown in Figure 8 for a typical violent manoeuvre, with filtered values again offset from unfiltered ones. The low cut-off frequency filter clearly distorts the acceleration response, while the high cut-off frequency filter does not distort the signal perceptibly. However, for the latter, while the main noise component at 17·5 Hz is effectively removed, significant noise is still present at 3·5 Hz. The transfer function coefficients and phase delay of each filter are given in Table 2.

TABLE 2

Specifications of Standard Digital Butterworth Lowpass Filters

Quantity	Filter 1 (low cut-off frequency)	Filter 2 (high cut-off frequency)
T (s)	1/60	1/60
N	5	5
f (Hz)	3·5	4·0
A (dB)	50·0	3·0
f_c (Hz)	1·12	4·0
Ω_c (rad/s)	7·0331292892882D+00	2·5518903306118D+01
τ (s)	0·459	0·127
E_0, F_0	1, 1·7478224133541D+06	1, 4·5580802280576D+03
E_1, F_1	5, −8·0769814534886D+06	5, −1·6621499551730D+04
E_2, F_2	10, 1·4953389527402D+07	10, 2·4877628218574D+04
E_3, F_3	10, −1·3862462739192D+07	10, −1·8991475064559D+04
E_4, F_4	5, 6·4345340695383D+06	5, 7·3691389324960D+03
E_5, F_5	1, −1·1962698176138D+06	1, −1·1598727628397D+03

5. CONCLUDING REMARKS

For the purpose of removing significant noise from flight data on a Sea King Mk. 50 helicopter, suitably designed digital Butterworth lowpass filters are demonstrated to be effective. For simplicity and economy of programming effort, the direct form of representation of the transfer function as a high-order filter is successfully implemented, using double precision to avoid the usual problem of numerical accuracy caused by high coefficient sensitivity. That no reference in the literature could readily be found in which the filter coefficients are derived in direct form for a general-order filter is probably because of the generally accepted practice of representing a high-order filter appropriately as a number of second-order and first-order sub-filters. Hence, the lengthy algebraic process is included here. Using representative flight data, spectral analysis is used to clearly identify unwanted frequency components. To accentuate these components, which are mostly at the rotor frequency of 3·5 Hz and higher harmonics, the signal components at low frequencies up to about 1 Hz are first removed by subtracting, from the unfiltered data, values obtained using the low cut-off frequency filter eventually adopted. Channels measuring control inputs and directly linked displacement quantities are not filtered because of the absence of significant noise and to avoid further distorting the signal. Two standard fifth-order filters are shown to be adequate for all filtering requirements. One, with a low cut-off frequency of 1·12 Hz, is suitable for variables where the energy is confined to relatively low frequencies, while the other, with a higher cut-off frequency of 4 Hz, is used alternatively where the high frequency content is significant. For both filters, adequate correction for phase shift is obtained on assuming a constant time delay.

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APPENDIX A

Fortran Subprogram Giving Transfer Function Coefficients in Direct Form for a Digital Lowpass Butterworth Filter

```
      SUBROUTINE TRANSF(N,SFREQ,OMEGAC,E,F,A,B)
C
C  CALCULATES NUMERATOR AND DENOMINATOR COEFFS OF TRANSFER FUNCTION
C  (I.E. ARRAYS 'E' AND 'F') THAT DEFINE A DIGITAL LOWPASS BUTTERWORTH FILTER
C  WITH 'N' POLES, ANALOGUE CUT-OFF FREQ 'OMEGAC', AND SAMPLING FREQ 'SFREQ'.
C  ARRAYS 'A' AND 'B', THOUGH USED ONLY LOCALLY WITHIN SUBN, ARE DIMENSIONED
C  EXTERNALLY.  THE TRANSFER FUNCTION IS IN THE DIRECT FORM
C
      PARAMETER PID2=1.570796326794897D0      ! PI/2
      INTEGER SFREQ             ! 1/T
      INTEGER BCOEFF,E,A,B,D
      INTEGER P,Q,R,PP,QQ,RR
      DOUBLE PRECISION OMEGAC,F,PROD,AR
      DIMENSION E(0:N),F(0:N),A(0:N),B(0:N)
C
      DO 210 P=0,N
      PP=P
      E(P)=BCOEFF(N,PP)        ! NUMERATOR COEFF
C
      F(P)=0D0
C
      DO 200 R=0,N
      RR=R
      PROD=1D0
      IF(R.EQ.0) GO TO 120
C
      DO 110 M=1,R
      PROD=PROD*DCOS((M-1)*PID2/N)/DSIN(M*PID2/N)
110   CONTINUE
120   AR=PROD/OMEGAC**R
C
      DO 130 Q=0,N
      A(Q)=0
      B(Q)=0
130   CONTINUE
C
      DO 140 Q=0,R
      QQ=Q
      A(Q)=BCOEFF(RR,QQ)*(-1)**Q
140   CONTINUE
C
      DO 150 Q=0,N-R
      QQ=Q
      B(Q)=BCOEFF(N-R,QQ)
150   CONTINUE
C
      D=0
      DO 160 Q=0,P
      D=D+A(Q)*B(P-Q)
160   CONTINUE
C
      F(P)=F(P)+AR*D*(2*SFREQ)**R      ! DENOMINATOR COEFF
200   CONTINUE
C
210   CONTINUE
C
      RETURN
      END
```

```
      INTEGER FUNCTION BCOEFF(N,K)
C
C  GIVES BINOMIAL COEFFS
C
      INTEGER FACT
C
      BCOEFF=FACT(N)/(FACT(K)*FACT(N-K))
      RETURN
      END


      INTEGER FUNCTION FACT(N)
C
C  GIVES FACTORIAL OF AN INTEGER, I.E. N * (N-1) * (N-2) * ... * 1.
C  FACTORIAL OF ZERO IS 1
C
      FACT=1
      IF(N.LT.2) RETURN
      K=1
      DO 110 I=2,N
      K=K*I
110   CONTINUE
      FACT=K
      RETURN
      END
```

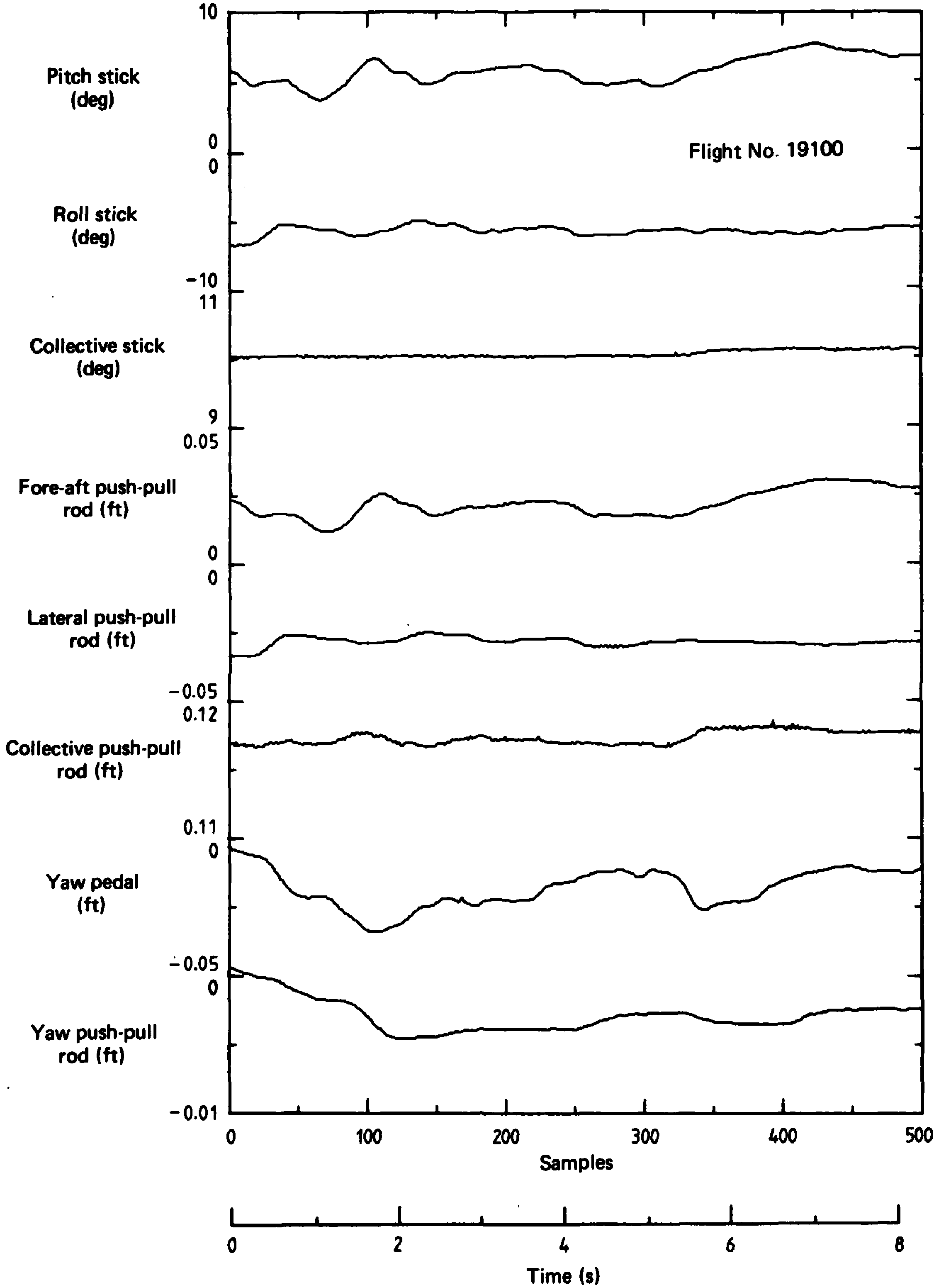

FIG. 1 UNFILTERED FLIGHT DATA – (1) CONTROL INPUTS AND DIRECTLY LINKED DISPLACEMENTS

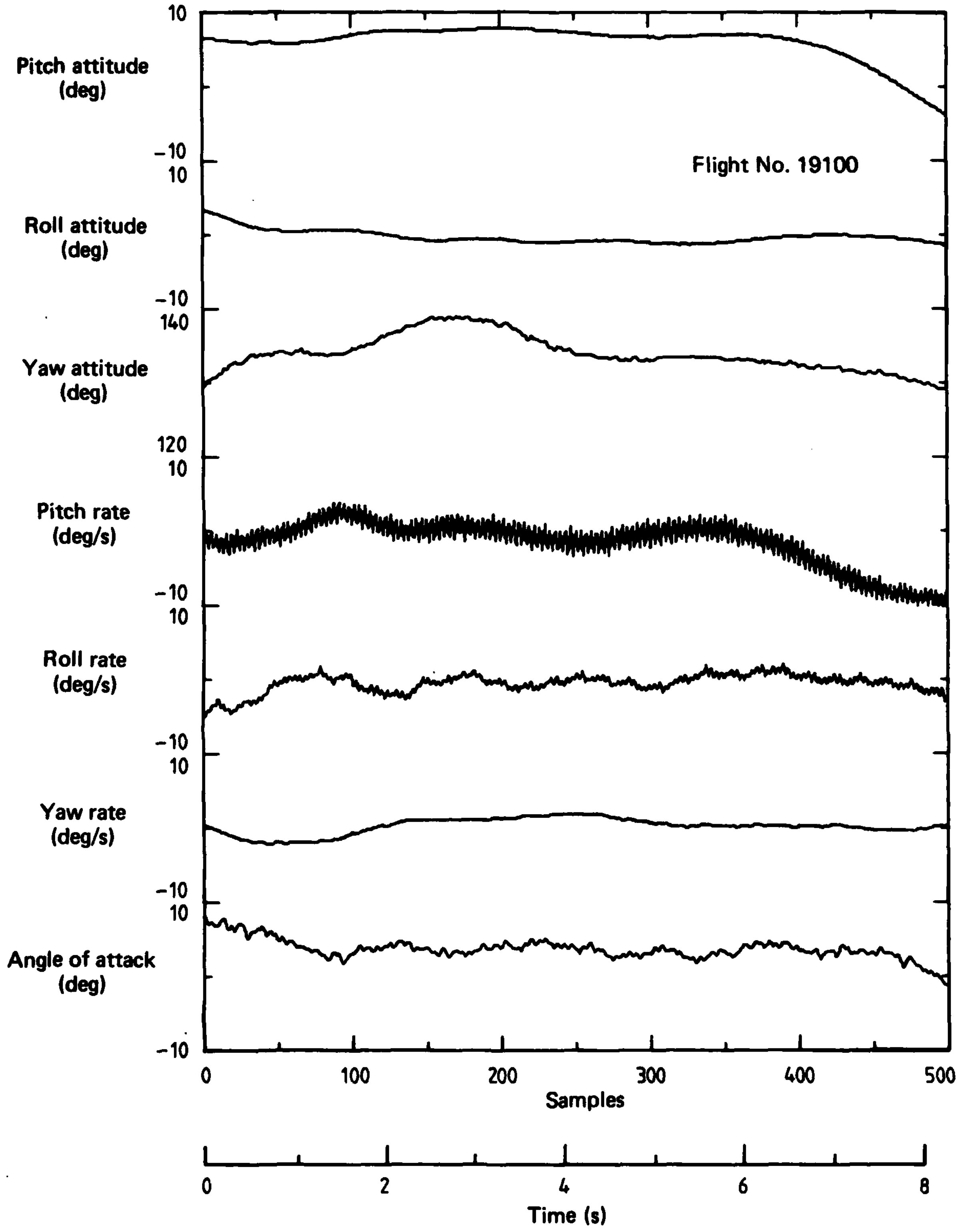

FIG. 2 UNFILTERED FLIGHT DATA – (2) ANGULAR DISPLACEMENTS AND VELOCITIES

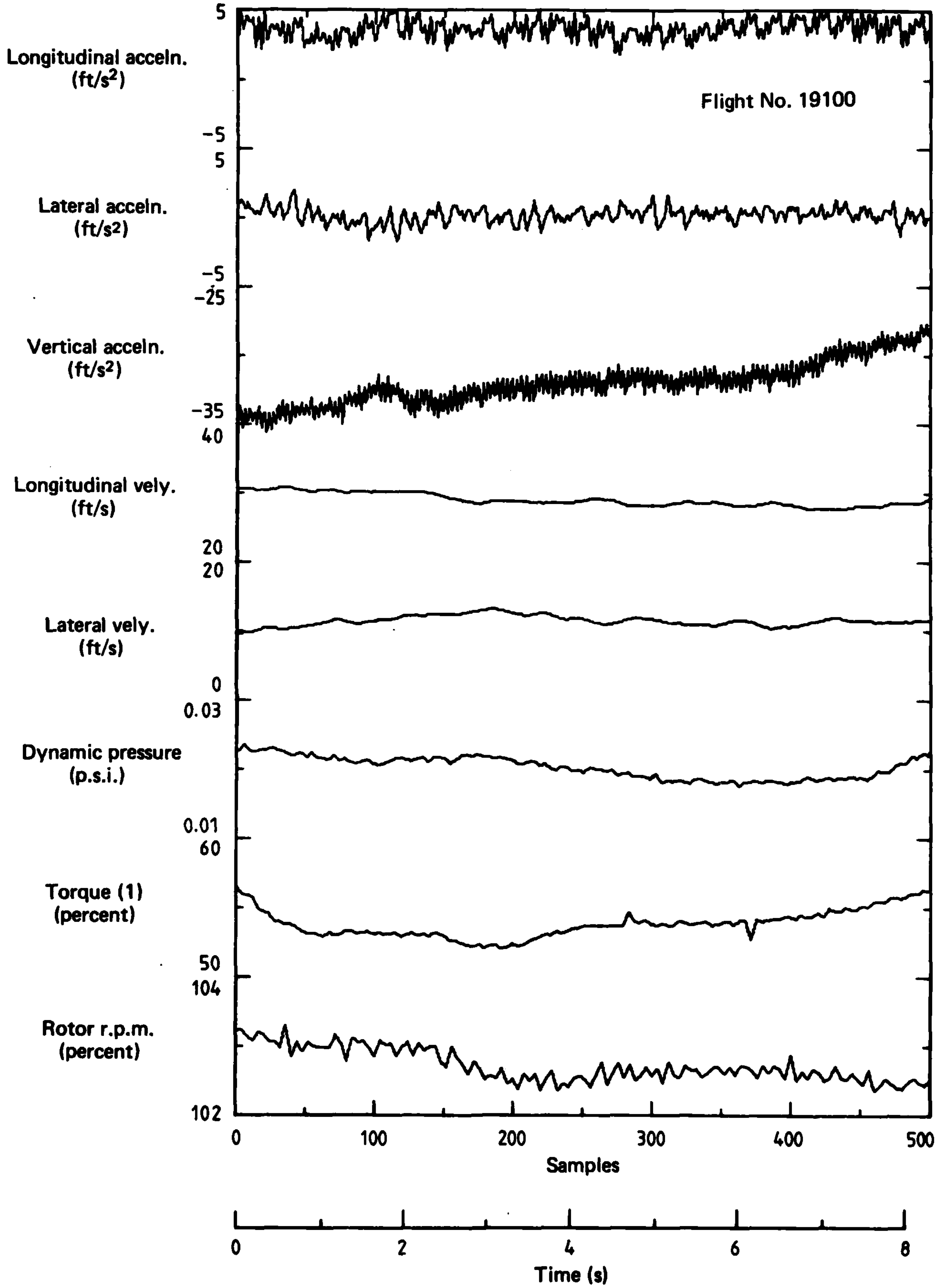

FIG. 3 UNFILTERED FLIGHT DATA – (3) LINEAR VELOCITIES AND ACCELERATIONS, AND MISCELLANEOUS QUANTITIES

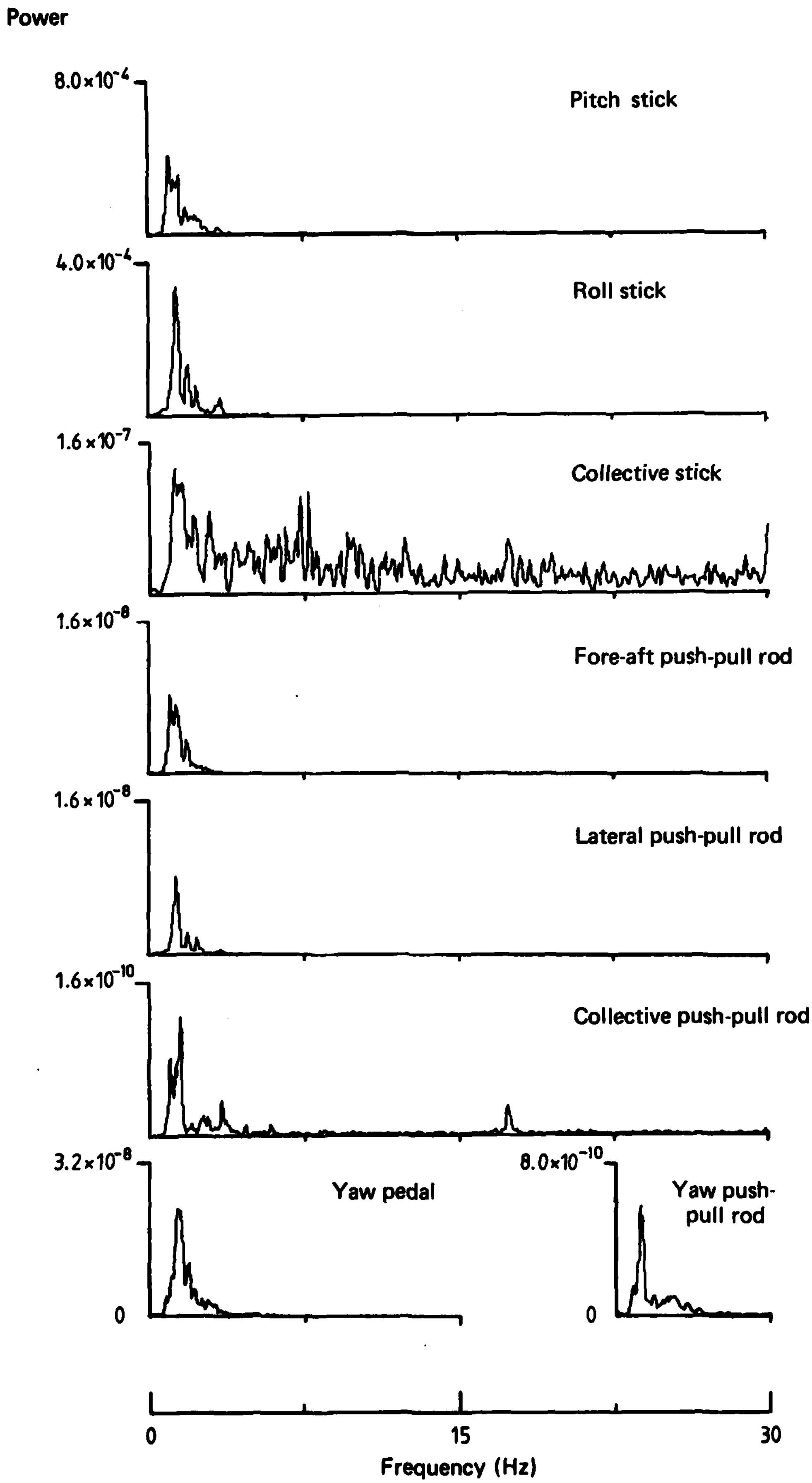

FIG. 4 NOISE POWER SPECTRA – (1) CONTROL INPUTS AND DIRECTLY LINKED DISPLACEMENTS

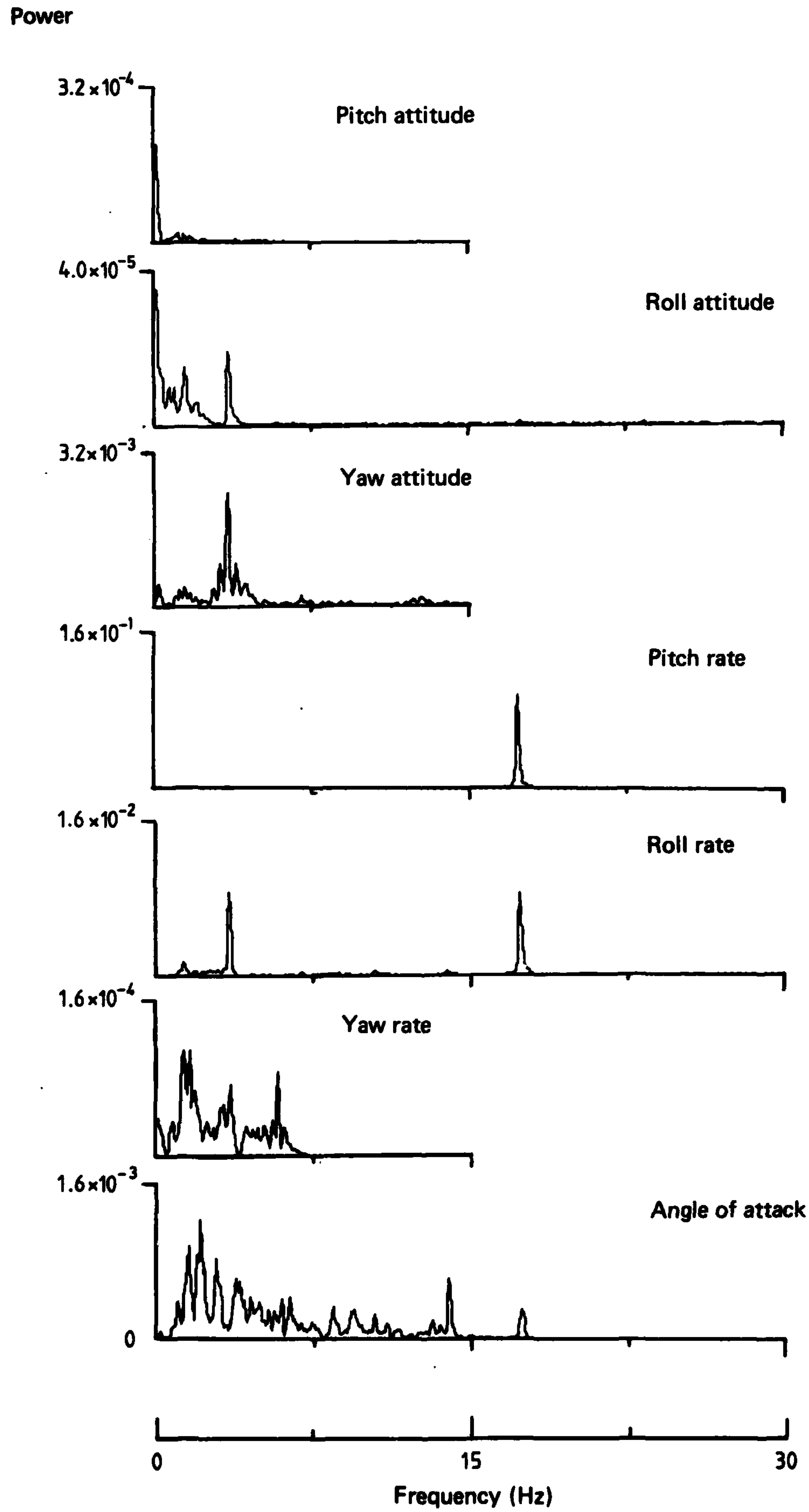

FIG. 5 NOISE POWER SPECTRA – (2) ANGULAR DISPLACEMENTS AND VELOCITIES

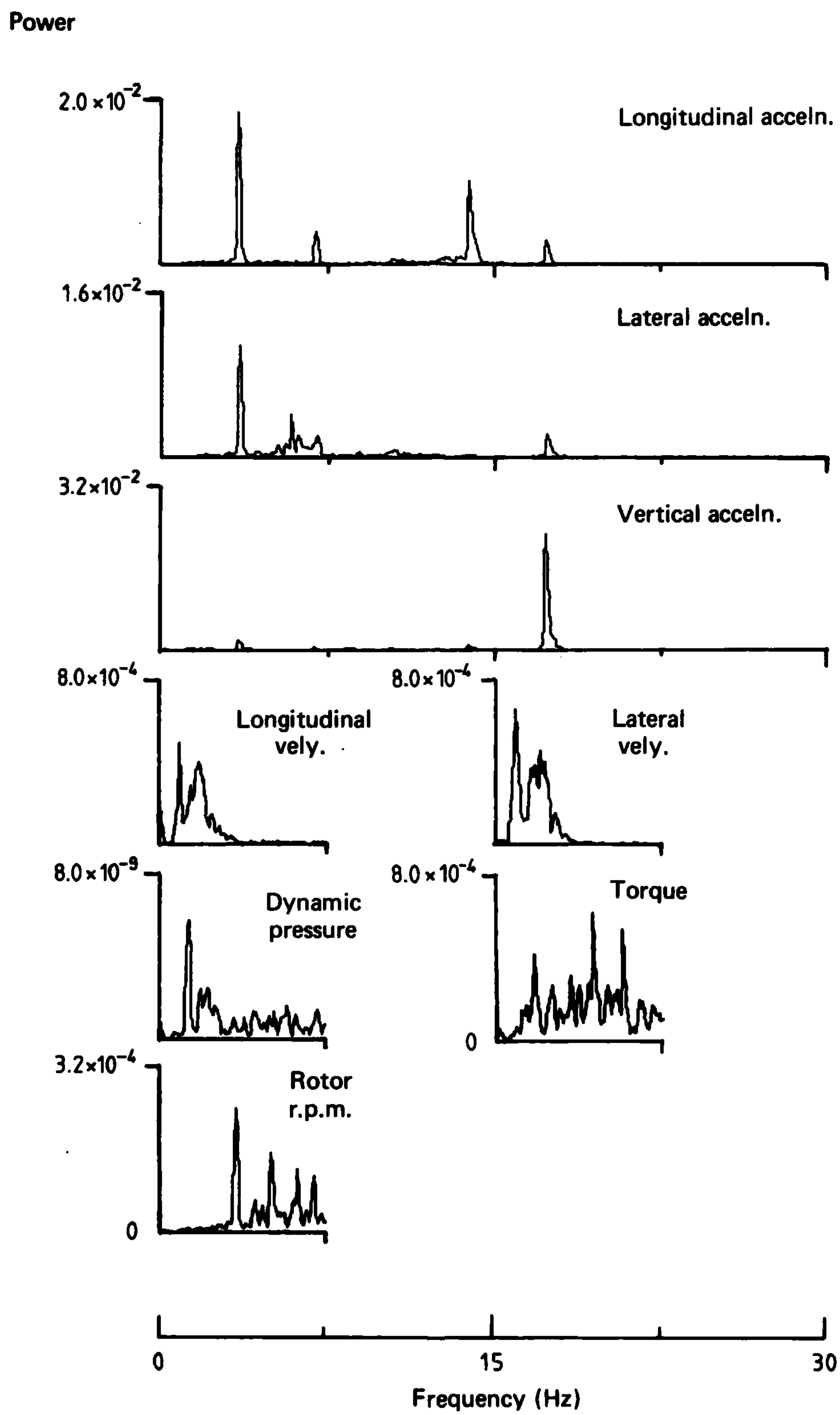

FIG. 6 NOISE POWER SPECTRA – (3) LINEAR VELOCITIES AND ACCELERATIONS, AND MISCELLANEOUS QUANTITIES

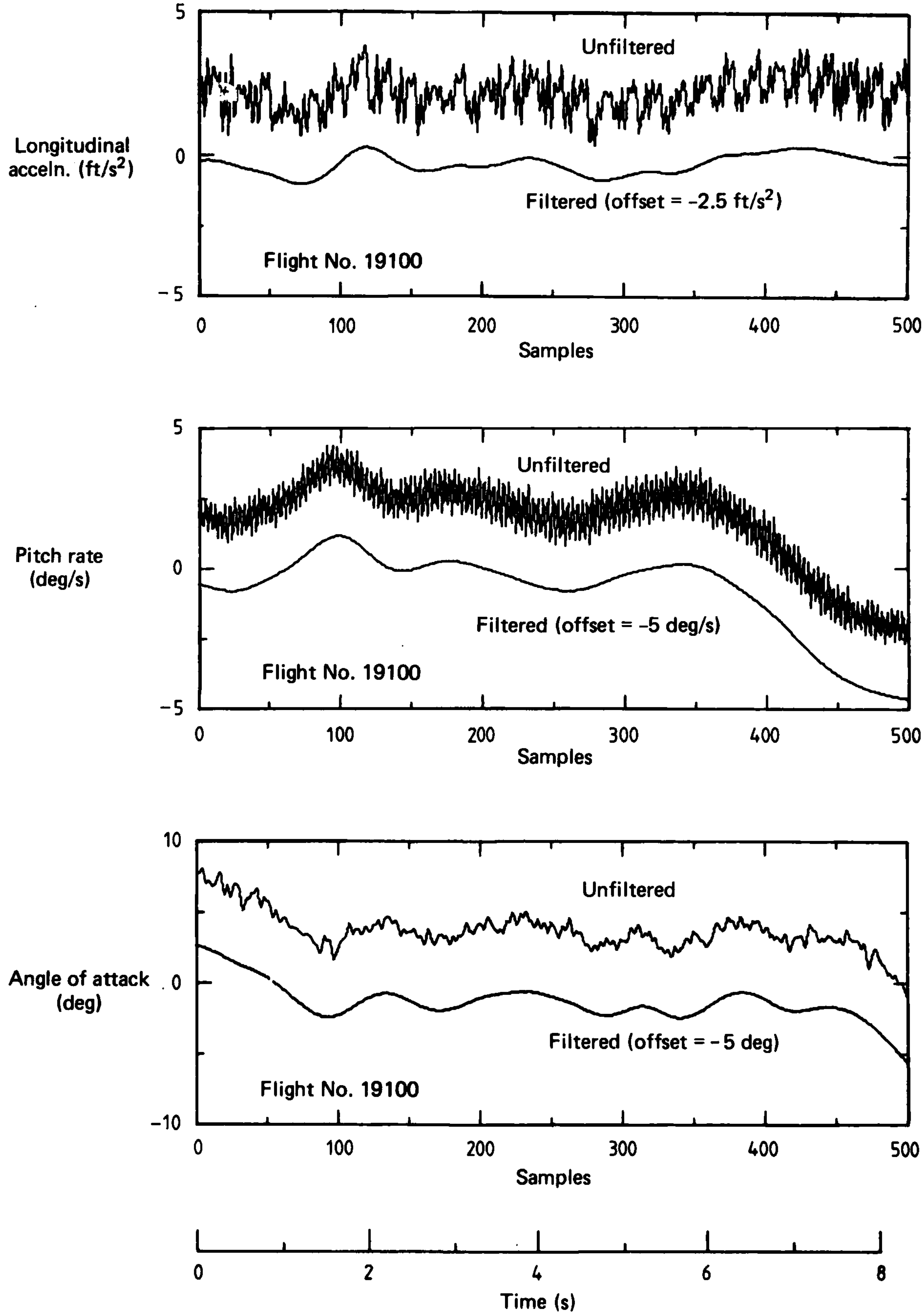

FIG. 7 EFFECT OF LOW CUT-OFF FREQUENCY (= 1.12 Hz) FILTER ON UNFILTERED FLIGHT DATA

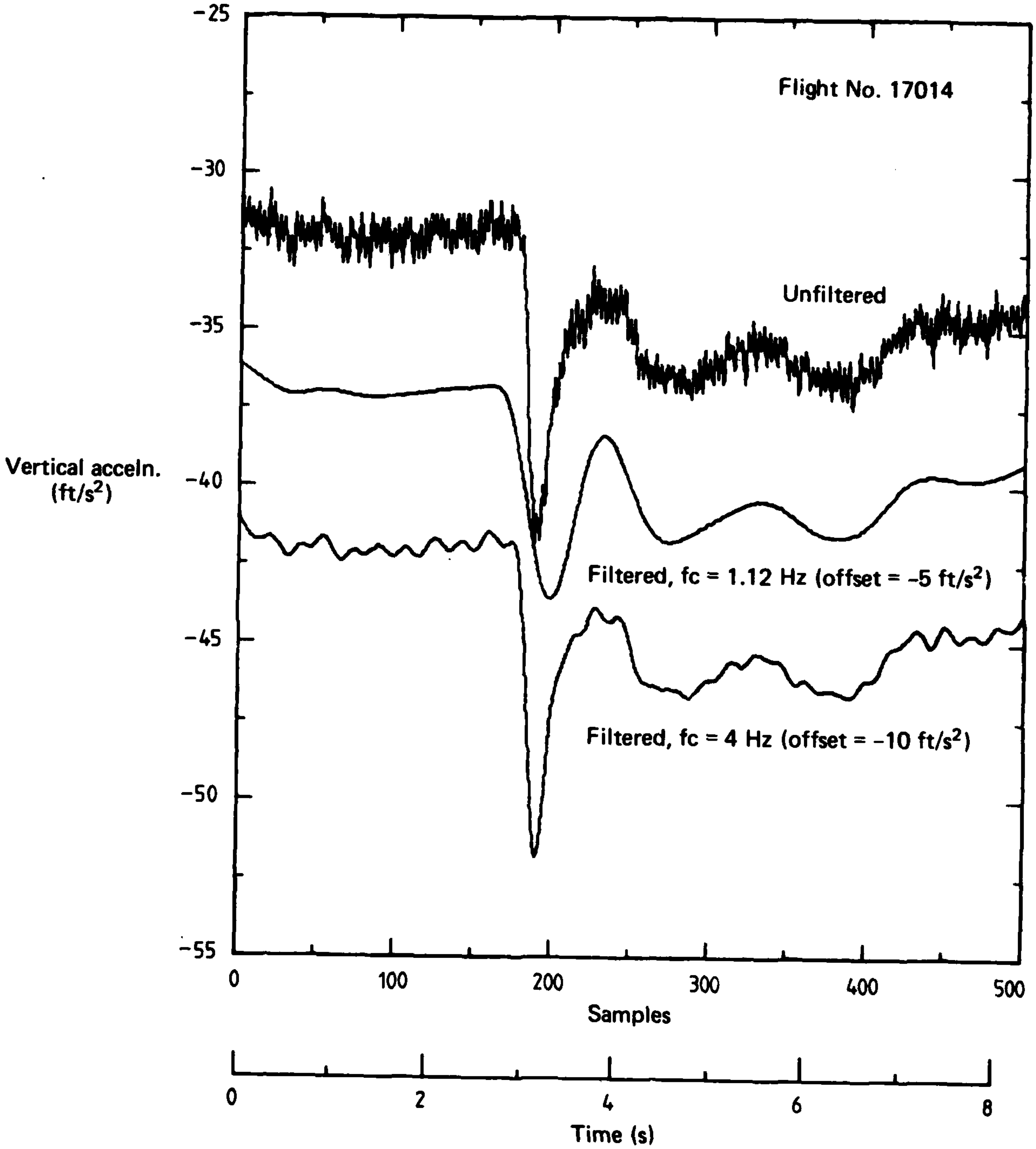

FIG. 8 EFFECT OF LOW AND HIGH CUT-OFF FREQUENCY FILTERS ON VERTICAL ACCELERATION SIGNAL IN REGION OF A SUDDEN CHANGE

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